GAUNTLET

Gauntlet Research Report

Market Risk Assessment

An analysis of the financial risk to participants in the Compound protocol.

Compound

February 2020

Contents

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Part I

Background

Compound allows participants to trustlessly supply and borrow Ethereum assets, providing appealing interest rates for borrowers and passive income for suppliers. By using collateral and amortizing risk across individual suppliers in a liquidity pool, Compound's Ethereum smart contract has been a profitable place to supply crypto since its inception in 2018. The protocol implemented in Compound's smart contract is detailed in the Compound whitepaper.

However, despite the fact that Compound has grown well past nine figures (of USD value) without any suppliers losing money, it is still technically possible, under extreme conditions, for borrowers to default on their borrowed assets and suppliers to lose their principal. Understanding when this failure condition can happen boils down to understanding various types of risks associated with the protocol, including protocol security risk,[1] governance risk,[2] and market risk. This report focuses on evaluating market risk — the risk of a user experiencing losses due to market fluctuations external to the smart contract itself.

We use a rigorous definition of market risks to construct simulation-based stress tests that evaluate the economic security of the Compound protocol as it scales to underwriting billions of dollars of borrowed assets. These stress tests are trained on historical data and put through a battery of scenarios that represent the expected and worst case economic outcomes for the protocol. Our stress tests are constructed analogously to how transaction-level backtesting is done in high-frequency and algorithmic trading. These techniques are used to estimate the market risk of a systematic trading strategy before it is deployed to the market. As there are over $1 trillion US dollars of assets managed by funds that use these techniques to provide daily actuarial analyses to risk managers, we believe that these are the best methodologies for evaluating market risk.[3] By modifying these techniques to handle the idiosyncrasies of cryptocurrencies, we are able to provide similar statistical power in these actuarial analyses.

The first portion of this report will define the set of market risks that users of the Compound protocol face, breaking them down into their principal quantitative components. Subsequently, we will describe the incentives behind the mechanism that the Compound protocol uses to ensure that it is solvent — liquidations. Finally, we will conclude by detailing how liquidators are similar to trading strategies and detail the market impact models that are used to analyze their incentives and expected returns.

The second portion will focus on methodology and results from agent-based simulations of the Compound smart contract. Our methodology utilizes careful simulation to closely replicate

[1] Examined by independent smart contract auditors: Certora, OpenZeppelin, and Trail of Bits

[2] More broadly, this refers to things like administrator mismanagement, voter participation, etc.

[3] Arnott et al. (2005); Curcuru et al. (2014); Hsu (2004)

the live environment that users interact with in the Compound protocol. This approach and some of our novel technologies, such as a custom Ethereum virtual machine, ensure that our results replicate reality with high fidelity. We conclude by detailing the results of these simulations, providing actuarial assurances for the conditions under which the Compound protocol is insolvent.

Our conclusions show that **the Compound protocol can scale to a larger size and handle high volatility scenarios for a variety of collateral types**. In particular, we find statistically significant evidence that even when Ether (ETH) realizes it's maximum historical volatility, the Compound system is able to grow total borrowed value by more than 10x while having a sub-1% chance of default.[4] Note that in this report we will refer to the protocol being in 'default' as equivalent to being under-collateralized. Moreover, we find that the system stays significantly over-collateralized in extreme scenarios and that current liquidation incentives are sufficient for more liquid collateral types, such as ETH. We also note that for collateral that realizes super-linear slippage (e.g. trading costs per unit quantity increase with larger liquidation sizes), one needs to be more aggressive with both liquidation incentives and collateralization ratios. The same techniques used to justify these conclusions also provide guidance on how to set protocol parameters for new collateral types that are added in the future. Finally, we note that more detailed descriptions of our simulation methodology and a glossary of terms utilized throughout this report can be found in Appendix 9.

[4] This 10x is relative to the size of the Ether market. In the case where that grows a commensurate amount, as it easily could, then Compound could grow even larger.

Part II

Defining Market Risks

1 Market Risks

The decentralized nature of the Compound protocol renders risk assessment both more complex and crucial than similar assessments in traditional markets. The main causes for this increase in complexity are the multitude of participant behaviors in the Compound protocol as well as their interactions with exogenous markets, such as centralized cryptocurrency trading venues. Unlike formal verification and smart contract auditing, which focus on *clbmec+lmsq* risks within a smart contract, economic analysis of protocols focuses on how *cvmeclmsq* shocks affect participant behavior. As the Compound protocol uses a deterministic function of liquidity supply and borrowing demand to determine the interest rates that suppliers and borrowers receive, one need only consider market prices, supplier supply behavior, and borrowing demand to accurately model exogenous risk (see Appendix 10). More specifically, the primary sources of exogenous risk stem from the following components:

1. Shocks to market prices of collateral that cause the contract to become insolvent due to under-collateralization

2. Loss of liquidity in an external market place, leading to a liquidator being disincentivized to liquidate defaulted collateral

3. Cascades of liquidations impacting external market prices which in turn lead to further liquidations (i.e. a deflationary spiral)

In order to quantify the effects of these risk components, we first need to delve into the notions of assets and liabilities within the Compound protocol.

1.1 Assets and Liabilities

In the Compound protocol, the main assets are the collateral tokens that suppliers have committed to liquidity pools, whereas the main liabilities are the outstanding borrowed assets. Token holders contribute their ERC-20 assets to a liquidity pool, and are in turn paid a yield on their supplied tokens. Borrowers borrow an asset by first committing collateral before withdrawing up to a certain amount from the liquidity pool. This amount is controlled by the *amj+j_rcp_j d_armp*,[5] which is the ratio of the maximum outstanding debt to collateral. The system

[5] You can find more information on the collateral factor in the Compound developer documentation.

forces borrowers to over-collateralize their borrowed assets (e.g. a fully-secured credit facility), thus enforcing the invariant that assets must always be greater than liabilities. For instance, one can deposit $100 of ETH[6] into the contract and withdraw $75 if the contract has a collateral factor of 75%. The borrower's collateral requirement is the value of outstanding debt divided by the collateral factor. When the value of the borrower's collateral asset falls below the collateral requirement, the collateral position becomes liquidatable.

The *net liabilities* of Compound are defined as the asset values less liabilities, so that the system is deemed solvent when the net liabilities are positive. As a decentralized protocol, Compound utilizes a series of economic incentives to ensure that net liabilities are always positive. When the market value of the collateral backing a lien falls below the collateral requirement, the protocol sells the collateral at a discount to a liquidator. This discount, termed the *liquidation incentive*, provides a liquidator with financial incentive to buy the collateral from the protocol, effectively repaying the borrowed asset on behalf of the borrower. With liquidation, the protocol acts much like a bank selling a defaulted asset at a foreclosure auction to increase their net liabilities. In particular, the liquidator acts analogously to the foreclosure auction winner, who is usually able to claim the defaulted asset at a discount.

As an oversimplified example, suppose that the Compound protocol has an ETH borrow position that is in default, with the current collateral amount equal to $100. If the liquidation incentive is 105% (5% extra bonus), then the liquidator would pay the Compound Smart Contract $95 for the ETH collateral. Moreover, if the liquidator has low time preference (Appendix 11.6.2), then they will sell the collateral as soon as possible. In practice, the Compound protocol only lets liquidators liquidate a portion of the borrow amount, and they receive collateral equal to 105% of the borrow value repaid. This has the benefit of increasing the collateralization ratio on the remaining portion of the borrowed asset, while avoiding complicated mechanics of completely closing borrow positions.[7] In this sense, liquidation in Compound resembles an algorithmic trading strategy, as there is a race to be the first liquidator to claim portions of the collateral and sell it on the market with minimal transaction and slippage costs.

1.1.1 Synthetic Assets: cTokens

There is a slight nuance in how assets and liabilities are treated — technically, the assets that suppliers and borrowers interact with are cTokens. These tokens, which wrap standard ERC-20 assets, serve as contingent claims on assets and earned interest. Suppliers supply assets

[6] In this stylized example, we use US Dollars as a numéraire, whereas in reality, one would have to execute this transaction in the Compound protocol against a USD stablecoin. Stablecoins are digital representations of US dollars, with some backed by bank deposits (USDC, TUSD) and others backed by digital collateral (DAI).

[7] Contrast this with the model MakerDAO uses, where there are auctions to liquidate the entire borrowed asset. This can create a delay which adds to market risk as well as unnecessarily closes borrow positions which could be merely reduced to a safe level.

as ERC-20 tokens and are returned cTokens, whereas borrowers supply collateral, which is converted to a cToken and used to make outstanding interest payments. Unlike traditional assets, cTokens immediately realize earned interest as payments are paid pro rata to holders on every block update.

Technically, there is a security risk that a cToken cannot be converted back to the underlying asset if the contract has many outstanding borrowed assets that are not being repaid as collateral is redeemed. This would mean that the contract is illiquid, but not necessarily insolvent. This report focuses on solvency, and liquidity will be considered more deeply in future analysis.

1.2 Risk Sensitive Parameters of the Protocol

The main levers protocol designers can wield in Compound to reduce risk are the collateral factor and liquidation incentive. However, these two levers impact the incentives of the protocol in different ways. The collateral factor controls the riskiness of borrowers — the closer it is to 100%, the more likely risky borrowers will default by borrowing USD stablecoin against collateral that is rapidly decaying in value. On the other hand, the liquidation incentive controls how likely liquidators are to take liabilities off of the smart contract's balance sheet. The higher the liquidation incentive, the less time a defaulted borrowed asset will be a liability on the Compound protocol. If we dissect how the three risk components of §1 connect to these two parameters, we find the following:

- The risk inherent in the collateral factor is connected to the nature of shocks to the market price of the collateral
- The risks that liquidators with low time preference face is connected to the loss of liquidity in an external market place
- Cascading liquidations affect both the collateral factor and the liquidation incentive because they create a feedback loop between price shocks and a loss of liquidity

This implies that under normal market conditions, when liquidations are independently distributed (e.g. uncorrelated), the collateral factor and liquidation incentive control borrower risk and supplier's ability to recoup losses, respectively. However, in situations when liquidations have a 'knock-on' effect and are correlated, these parameters affect both borrower and supplier behavior. Therefore, to study the true market risk of the system, we need to sample a variety of market and liquidity conditions in order to stress test these scenarios.

2 Liquidation

Akin to foreclosure sale participants in traditional finance, liquidators can repay the outstanding debt with discounts in exchange for the borrower's cToken collateral. In both foreclosure sales and in Compound liquidations, discounts are used to incentivize purchases of defaulted collateral. The Compound protocol provides a discount by giving liquidators additional collateral as the liquidation incentive to perform liquidation. However, unlike the all-or-nothing transactions of foreclosure sales, an individual liquidator can only repay a portion of the debt. The *ajmqc d_armp* is the protocol parameter that specifies the proportion eligible to be liquidated by any individual liquidator. When a liquidator finds a profitable trade, she repays a portion of the outstanding debt (determined by the close factor) in return for the borrower's collateral. Depending on a liquidator's risk preference, she may sell the collateral immediately to protect against price-fluctuation risk or just hold the received collateral.

Liquidation incentives create an arbitrage opportunity or a price discount for the liquidator in exchange for the reduction of Compound's risk exposure. The higher the liquidation incentive is, the more liquidators will participate in the liquidation process as they get steeper discounts relative to market prices. In other words, tuning the liquidation incentive is one of the most effective ways to adjust the protocol's safety boundary. The liquidation incentive also has an influence on a borrower's decision to borrow asset within the protocol. When a borrower's lien is liquidated, the liquidation incentive can be viewed as a bonus amount of a borrower's collateral that is given to the liquidator to compensate for the risk they engender while taking a liability off of the protocol's balance sheet. If the liquidation incentive is too high, a borrower may be unwilling to borrow assets from Compound in the first place, or she may open a borrowing position and maintain a high collateral factor. In general, one expects that increased liquidation incentives negatively impact borrowing demand.

The collateral factor defines a maximum borrowing capacity for each asset enabled within the protocol. Borrowers must manage their own debt and keep their liens over-collateralized to ensure a certain margin of safety with respect to the maximum borrowing capacity. This margin of safety fluctuates with market conditions and depends on the borrowers' own risk profile. When the market volatility is high, risk-averse borrowers maintain a high margin of safety to avoid their collateral being liquidated. In contrast, risk-seeking borrowers maintain a low margin of safety and actively refinance their debt to optimize their usage of borrowed capital. Understanding the interaction between collateral factor and the safety margin requires studying the influence of psychology on the participant's behavior. Randomized controlled trials and other experimental methods are designed to understand this type of causal relationship.

Rational liquidators with short time preference are defined to be participants who purchase collateral from the Compound smart contract and immediately sell it on a centralized venue (e.g. have low risk tolerance). For brevity, we will refer to rational liquidators with short time preference as *epccbw jgosgb_rmpq*. To simplify the analysis and simulate the worst-case scenario for Compound, we assume that all liquidators are greedy and sell the collateral immediately to a market, instead of having liquidators that repay the outstanding debt and hold the collateral. This focus on greedy liquidators emulates the worst-case protocol behavior as adverse market and liquidity conditions can cause cascading defaults. Greedy liquidators tend to inflame cascading defaults as they create sell pressure and can cause a deleveraging spiral.[8] The main source of loss for greedy liquidators is the loss due to price impact, or *qjgnn_ec*, that is caused by selling a large quantity of an asset. Given that greedy liquidators immediately sell, they must optimize the quantity that they are willing to liquidate based on market prices and expectations of slippage.

3 Slippage

Slippage refers to the expected change in a tradeable asset's price p due to a matched order of size q and is mathematically denoted $p(q)$. Formally, $p(q)$ is defined to be the difference between the market midpoint price and the actual average execution price when a market participant executes a trade. Slippage inevitably happens on every trade, and this effect tends to be magnified in thin or high volatility markets. For a liquidation opportunity, slippage is the only cost that can be partially controlled by the liquidator, whereas trading fees and smart contract transaction fees are usually external restrictions. Therefore, slippage is one of the major factors that influence a liquidator's decision-making.

Market impact, which is a synonym for slippage, has been studied extensively in traditional finance.[9] Many market impact models have been proposed and tested for solving optimal order execution problems. In traditional markets, the marginal increase in price impact is usually observed to decrease as a function of trade quantity, which formally corresponds to $p(q)$ being a concave function.[10] However, this appears to not be true for cryptocurrency markets, where empirical data suggests that $p(q)$ is linear or even convex (e.g. the marginal cost *g/+ apc_qcq* with quantity).[11] Despite each type of model having different underlying assumptions and functional forms, a majority of the models comprise trade volume-to-market size, volatility and time variables. Analyzing trade size, volatility and how these variables interact with liquidation is the primary focus of this analysis. The analysis in this report only considers greedy liquidators that sell repossessed collateral on centralized exchanges with order books, such as Coinbase and Binance. As decentralized exchanges and automated market makers, such as

8 See Klages-Mundt and Minca (2019) for an in-depth discussion of this in MakerDAO

9 Tóth et al. (2011); Gatheral and Schied (2011)

10 Eisler et al. (2012); Gatheral (2010)

11 Makarov and Schoar (2019); Wei (2018)

Uniswap,[12] provide an alternative source of liquidity, one might ask why this assumption was enforced. The reasons for this choice are two-fold:

- Order book depth on centralized exchanges is order of magnitudes greater than that of decentralized exchanges for most assets[13]
- Slippage in automated market makers is usually designed to be small for small trades and expensive for large quantities, so greedy liquidators would likely end up going to a centralized exchange during the most volatile times to stay profitable

We will break up the dominant features of slippage into *k_picr t_pg_`jcq* that are exogenous to the Compound smart contract state and *npmrmamj t_pg_`jcq*.

3.1 Key Market Variables

3.1.1 Outstanding Debt

The total traded quantity that the protocol will need liquidated in times of net negative liabilities will be a function of the total outstanding debt in the system. Since this quantity is the input to the slippage function $p(q)$, it is clear that the choice of slippage model needs to be cognizant of the amount of outstanding debt. We will define the amount of outstanding debt in this analysis to be the sum of all the borrowers' total outstanding debt value normalized by the average daily trading volume of underlying collateral. This metric captures the size of debt relative to the underlying liquidity, and gives readers a good intuition around how big Compound's market can grow safely relative to the trading markets. Since the trading volume of different assets varies, using unitless metrics (such as the amount of outstanding debt) provides a more intuitive comparison between different assets. The simulation in this report assumes borrowers borrowing USD stablecoin backed by ETH, as this is the most common use case in the Compound protocol. As an example, suppose that the ETH daily trading volume is 100 million USD, 0.5 total outstanding debt is equivalent to 50 million USD of total outstanding debt value.

Estimating the average daily trading volume of cryptocurrencies is difficult, as wash trading and other market manipulation practices are known issues in the cryptocurrency market.[14] Numerous studies have concluded that the reported volume from various cryptocurrency exchanges may be unrepresentative of the assets' underlying liquidity. For this reason, we aggregated the average daily trading volume from the top 10 exchanges with well-functioning

[12] Angeris et al. (2019)

[13] We do note that this is not true for assets such as MKR and SNX, as their primary market is Uniswap. However, for the larger assets that are listed on Compound such as ETH, DAI, and REP, there is far more centralized exchange liquidity.

[14] Alameda Research (2019)

markets identified by Bitwise Investments.[15] This indexing methodology has been adopted as the de facto industry standard, with major brokers and the Securities and Exchange Commission utilizing the Bitwise index for volume estimation.[16]

3.1.2 Asset Volatility

Volatility measures the degree of variation of asset price changes over a given time interval. Historically, it is traditionally defined as the standard deviation of logarithmic returns and is usually denoted .[17] Research studies show that volatility is typically a linear coefficient in a market impact model.[18] Given that asset volatility changes over time and is affected by market microstructure, it's equally important to understand how liquidator behavior changes when the market volatility changes. We assess this by sweeping through a variety of different volatility levels to ensure that we emulate how greedy liquidators interact with a plethora of market environments. Note that we normalize our volatility calculation in a manner akin to what is used by exchanges such as BitMEX.[19]

3.2 Key Protocol Variables

3.2.1 Liquidation Incentive

The liquidation incentive is the main driver for liquidators to repay borrowers' outstanding debt. If liquidation incentives don't exist, no rational liquidator will be willing to reduce the borrower's risk exposure during the collateral price drop. The size of liquidation incentive has a substantial influence on the liquidators' decision-making process. A greedy liquidator adjusts their strategy to ensure that she receives positive returns on every liquidation opportunity. Borrowers' outstanding debt will no longer get repaid if the liquidation incentive is too low and can't cover the cost of arbitrage, which includes slippage, trading fees, and transaction fees. Note that latency costs and smart contract front-running[20] are intentionally left out of this analysis because these introduce additional complexity. In particular, we use constant gas costs throughout all simulations detailed in this report. We can adjust our simulations to handle front-running, should there be further empirical evidence that liquidations costs are dominated

[15] Hougan et al. (2019); Bitwise Asset Management (2019)

[16] Securities and Exchange Commission (2019)

[17] Hull (1991)

[18] Mathematically, this means that there exists a function $f : [0, \infty) \to \mathbb{R}$ such that $p(q) = \; f(q) + o(1)$; see Hull (1991); Almgren et al. (2005) for theoretical and empirical evidence of this. In particular, note that this appears to hold for many markets in terms of ncpk_lclr impact cost, whereas instantanous impact cost tends to depend much more on an asset's microstructure details.

[19] BitMEX (2020)

[20] Daian et al. (2020)

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/ZTUSVQ FU BM

$PNQPVOE NBSLFU 8IFO BO BTTFU QSJDF ESPQT BOE OP MJR
UIF CPSSPXFS T PVUTUBOEJOH EFCU UIF TZTUFN GBJMT BOE
BTTFUT 3FDBMM UIBU B SBUJPOBM MJRVJEBUPS T HPBM JT UI
XIJDI EFQFOET PO UIF MJRVJEBUJPO JODFOUJWF BOE TMJQQ
BOE WPMBUJMJUZ *O MJHIU PG UIJT UIF NBJO RVFTUJPOT U

s *T UIF QSPUPDPM TBGF XIFO UIF UPUBM PVUTUBOEJOH EF

s *T UIF QSPUPDPM TBGF VOEFS WPMBUJMF NBSLFU DPOEJU

s *G $PNQPVOE XBOUT UP TVQQPSU B OFX BTTFU IPX TIPVMI
BOE DPMMBUFSBM GBDUPS TP UIBU UIF TZTUFN XJMM IBWF

8F XJMM GJSTU EFGJOF TPNF NFUSJDT UIBU XJMM IFMQ VT B
NBOOFS VOEFS DPMMBUFSBM BTTFUT JO MJRVJEBUJPO SVOPUG BUUFOE BT XFUUPIG
UIF NBSLFU T UPUBM PVUTUBOEJOH EFCU VOEFS DPMMBUFSBM
SVO QFSDFOUBHF DFE BT UIF QFSDFOUBHF PG TJNVMBUJPO SV
SVOT 5IJT NFUSJD JT VTFE UP RVBOUJGZ UIF TBGFUZ PG UIF
JG CPSSPXFST XJUI B MBSHF BNPVOU PG PVUTUBOEJOH EFCU
UP FOTVSF UIBU UIF TZTUFN JT OFWFS VOEFS DPMMBUFSBMJ
EFGJOF UIF GBJMVSF DSJUFSJB
5IF TJNVMBUJPO BTTVNFT CPSSPXFST VTF &5) BT DPMMBUF
%"* GSPN UIF $PNQPVOE QSPUPDPM &BDI MJRVJEBUPS FWBM
SBUJP BOE SFQBZT %"* PO UIF CPSSPXFS T CFIBMG JG UIFSF
UFTUFE B XJEF SBOHF PG NBSLFU DPOEJUJPOT BOE BOBMZ[F
TDFOBSJPT JODMVEF

s .PEFMJOH IJTUPSJDBM EBUB

s 4JNVMBUJOH WBSJPVT UPUBM PVUTUBOEJOH EFCU BOE B

s 4JNVMBUJOH WBSJPVT UPUBM PVUTUBOEJOH EFCU BOE M

s 4JNVMBUJOH WBSJPVT UPUBM PVUTUBOEJOH EFCU BOE D

3FTVMUT

)JTUPSJDBM %BUB

5IJT TFDUJPO HJWFT B CSJFG PWFSWJFX PG UIF MJRVJEBUJP
LFZ NFUSJDT DIBOHF PWFS UJNF 8F SFQMBZFE UIF QSJDF US

B &5) QSJDF USBKFDUPSZ C 8FJHIUFE BWFSBHF PG DPMMBUFSBMJ[BUJP

D #PSSPXFST DPMMBUFSBMJ[BUJPO SBUJP-JRVJEJUZ NFUSJDT

'JHVSF -JRVJEBUJPO NFDIBOJTN TJNVMBUJPO XJUI UIF QSJDF US
UPSZ 5IF TJNVMBUJPO BTTVNFT ..64% PG &5) EBJMZ U
UIF UPUBM PVUTUBOEJOH EFCU WBMVF JT ..64%

IJTUPSZ BOE TJNVMBUFE UIF MJRVJEBUJPO NFDIBOJTN 5IJT
UIF &5) QSJDF ESPQQFE 5IF TJNVMBUJPO XJMM SFTVMU JO SFEJ
JEFB PG XIBU JT JOWPMWFE B OUIF TJ[F PG MJRVJEBUJPO *O QSJDF UVSOT
PG MJRVJEBUJPOT "T UIF QSJDF ESPQQFE CPSSPXFST XJUI
GJSTU 8IFO UIF QSJDF CPUUPNFE PVU BU BCPVU B MBSH
HPU MJRVJEBUFE
'JHVSF C TIPXT UIF TJNVMBUFE MJRVJEJUZ QPPM T XFJHIUFE B
*O UIJT SFQPSU CPSSPXFST VTF &5) BT DPMMBUFSBM BOE XJ
QPPM "TTVNJOH UIBU UIF QSJDF PG %"* JT TUBCMF UIF DPM

JNQBDUFE CZ UXP GBDUPST &5) QSJDF BOE MJRVJEBUJPO &
WBMVF XIFSFBT MJRVJEBUJPOT SFEVDF UIF RVBOUJUZ PG %
8F SBOEPNMZ TBNQMF UIF CPSSPXFST JOJUJBM DPMMBUFSB
EJTUSJCVUJPO TF 5IF QJOFJOE BY M DPMMBUFSBMJ[BUJPO SBUJP
EFDSFBTFT BT UIF QSJDF EFDMJOFT 5IF MJRVJEBUPS IPMET
= 1/0 :75 EFGBVMU DPMMBUFSBM GBDUPS NJOJNVN DPMMBUFS
EFCUT XIJDI QSPWFT UIBU UIF MJRVJEBUJPO NFDIBOJTN JT G
*O GJHVSF D XF IJHIMJOF SFQSFTFOUT BO JOEJWJEVBM CPSSPXFS
TJ[F SFQSFTFOUT UIF CPSSPXFS T PVUTUBOEJOH EFCU WBM
USBKFDUPSZ XIJDI JT ESJWFO CZ UIF &5) QSJDF DIBOHF 8IF
UJPO PG UIF CPSSPXFS T DPMMBUFSBM BOE EFCU XJMM HFU S
DPMMBUFSBMJ[BUJPO SBUJP 4JNJMBS UP UIF QSFWJPVT DIB
SBUJP TIPVME OFWFS HP CFMPX NJOJNVN DPMMBUFSBMJ[
EBUJPO NFDIBOJTN *G B CPSSPXFS EPFT OPU HFU MJRVJEBU
CVrFS SFMBUJWF UP UIF CPSSPXFS T PVUTUBOEJOH EFCU W
VOEFS DPMMBUFSBMJ[FE
5IF MJRVJEJUZ NFUSJDT D BJ RVJE JUZ TIPSUGBMM JT EFGJO

DPMMBUFSBM WBMVF PVUTUBOEJOH EFCU WBMVF

&JUIFS MJRVJEJUZ PS TIPSUGBMM JT OPO [FSP EFQFOEJOH
UP UIF PVUTUBOEJOH EFCU WBMVF "T UIF &5) QSJDF EJQT U
CPUI USFOE EPXO 5IF TIPSUGBMM SFNBJOT BU TJODF BMM
MJRVJEBUFE JO UJNF

5PUBM 0VUTUBOEJOH %FCU WT "TTFU 7PMBUJM

"T XBT EJTDVTTFE UIF DPMMBUFSBM BTTFU T RVBOUJUZ UP
WPMBUJMJUZ BSF UXP NBKPS NBSLFU WBSJBCMFT DBVTJOH
GBDUPST JOGMVFODJOH B MJRVJEBUPS T CFIBWJPS 5IJT T
EFQFOET PO UIF UPUBM PVUTUBOEJOH EFCU BOE UIF DPMMBU
*O PVS TJNVMBUJPO UIF UPUBM PVUTUBOEJOH EFCU JT EFG
TUBCMFDPJO EFCU WBMVF OPSNBMJ[FE CZ UIF DPMMBUFSBM
UIBU EJrFSFOU DPMMBUFSBM BTTFUT IBWF EJrFSFOU PSEFST
J[JOH UIF UPUBM PVUTUBOEJOH EFCU FOBCMFT VT UP JOUVJ
DPMMBUFSBM BTTFU T MJRVJEJUZ CFUXFFO EJrFSFOU DPMM
B EBZ IFODF XF VTF EBJMZ WPMBUJMJUZ JOTUFBE PG DPNNP
TUSBJHIUGPSXBSE UP VOEFSTUBOE

5IFSF JT OPU TUSPOH BHSFFNFOU PO UIF EBJMZ USBEJOH W
USBMJ[FE FYDIBOHFT BSF TVTDFQUJCMF UP XBTI USBEJOH B
CZ UIFJS DFOUSBMJ[FE DPVOUFSQBSUT "T UIF BCJMJUZ UP T
GBDUPST PG TBGFUZ UIJT DSFBUFT BO VODFSUBJOUZ UIBU J
SBUJP PG PVUTUBOEJOH EFCU UP NBSLFU TJ[F XJEFMZ JO PVS
TDFOBSJPT UIBU ZPV NJHIU TFF JO UIF QSBDUJDF *G ZPV IBW
UIF UPUBM NBSLFU EFQUI PG UIF DPMMBUFSBM PSEFS CPPL Z
FYBNJOFE JO UIF UPQ PG UIF IFBUNBQ 0VS BTTVNQUJPOT PO
UJWF NN GBMMJOH PO UIF MPXFS FOE PG .FTTBSJ T EBJM

"MUIPVHI UIF QSPUPDPM QSPWJEFT MJRVJEBUJPO JODFOU
BWPJEBCMF DPTUT GPS B MJRVJEBUPS UP BSCJUSBHF JODM
GFF 5SBOTBDUJPO GFFT BSF UIF HBT GFFT QBJE UP BO &UIFS
BDUJPOT 8IFO B MJRVJEBUPS TFMMT IJT SFDFJWFE DPMMBU
FYDIBOHF

*O 'JHVSF XF TFF MJRVJEBUPS QSPGJU BOE MPTT DIBSUT CSPL
GFF TMJQQBHF BOE QSPGJU 5IFSF BSF NPSF BSCJUSBHF P
IJHI GJHVSF C BOE TVCTFRVFOUMZ UIF MJRVJEBUPS T UPUBM SFW
JT IJHIFS UIBO UIF SFWFOVF JO UIF B 0VS TJNVMBUJPO TIPXT SFNBJO
TMJQQBHF NPEFM XIJDI NFBOT UIBU UIF QSJDF TMJQQBHF J
TFF "QQFOEJY UIF FNQJSJDBM SBUJPOBMF 5IF DIBSU EFNP
JT UIF NBKPS DPTU PG BSCJUSBHF *O UIF IJHI WPMBUJMJUZ
PG UIF MJRVJEBUPS T SFWFOVF &WFO JO UIF MPX WPMBU
QBZ NPSF UIBO PG UIF SFWFOVF GPS TMJQQBHF *O CPUI TD
QFSDFOUBHF PG SFWFOVF BOE UIF PO DIBJO USBOTBDUJPO G
PVUTUBOEJOH EFCU

'JHVSF TIPXT UIF UPUBM MJRVJEBUFE EFCU BNPVOU XJUI EJrF
BOE &5) WPMBUJMJUZ 5IF SFTVMUT NBUDI PVS JOUVJUJPO U
UJPOBM UP CPUI UPUBM PVUTUBOEJOH EFCU BOE WPMBUJMJU
DPMMBUFSBM WBMVF IBT B IJHI DIBODF UP GBMM CFMPX UIF
RVFODF UIF DPMMBUFSBM XJMM HFU MJRVJEBUFE 5IPVHI MJ
QPVOE QSPUPDPM UIFZ DBO BMTP TFSWF BT B MFBEJOH JOEJ

'JHVSF EFNPOTUSBUFT UIF MJRVJEBUBCMF BOE VOEFS DPMM
3FDBMM UIBU BO BDDPVOU CFDPNFT MJRVJEBUBCMF JG UIF I
SFRVJSFNFOU BOE UIF DPMMBUFSBM JT BWBJMBCMF UP CF M
BTTFU ESPQT GVSUIFS BOE UIF DPMMBUFSBM WBMVF JT CFMP
CFDPNFT VOEFS DPMMBUFSBMJ[FE)FSF XF TFU B TUSJDU

.FTTBSJ)JTUPSJDBM &UIFSFVN %BUB

B EBJMZ WPMBUJMJUZ

C EBJMZ WPMBUJMJUZ

'JHVSF .FBO BHHSFHBUF MJRVJEBUPS QSPGJU BOE yDJPTTPUT PWFS
UIF MFGU IBOE TJEF JT VTJOH B MJOFBS TDBMF JO EPMMBST XIFS
TDBMF 5IF TJNVMBUJPO BTTVNFT .. 64% PG &5) EBJMZ USBEJO
PVUTUBOEJOH EFCU WBMVF JT .. 64%

DSJUFSJB J F UIF TJNVMBUJPO SVO GBJMT XIFO PWFS PG U
DPMMBUFSBMJ[FE 'PS FBDI EBUB QPJOU JO UIF IFBUNBQ XF
TBNF NBSLFU WBSJBCMFT BOE DBMDVMBUF UIF QFSDFOUBHF
QPJOU JT UIF GFXFS TJNVMBUJPO SVOT GBJM *G UIF EBUB Q
PG UIF TJNVMBUJPO SVOT IBWF NPSF UIBO PG UIF VOEFS DP
UP RVBOUJGZ UIF TBGFUZ PG UIF QSPUPDPM

5IF IFBUNBQT EFNPOTUSBUF IPX MBSHF UIF QSPUPDPM DBO
BTTVNQUJPO 5IF #JU.&9 XFFLMZ IJTUPSJDBM &5) WPMBUJMJ

'JHVSF 5PUBM MJRVJEBUFE EFCU BNPVOUT PWFS IPVS QFSJPE PG &5) EBJMZ USBEJOH WPMVNF " UPUBM PVUTUBOEJOH EFCU J UPUBM PVUTUBOEJOH EFCU WBMVF 5P FYQMBJO BO FYBNQMF DFM

PG UIF 5PUBM 0VUTUBOEJOH %FCU JO UIF DBTF XIJDI JT JOUV)PXFWFS UIJT EPFT NBUDI JOUVJUJPO CFDBVTF ZPV POMZ TFF UIJ WPMBUJMJUZ DMPTF UP

XFFLMZ WPMBUJMJUZ JO IJTUPSZ IBQQFOFE EVSJOH EBJMZ WPMBUJMJUZ JUZ"TTVNJOH UIBU UIF &5) NBSLFU DBQJUBMJ[BUJPO XJMM H EFDSFBTF XF DPOTJEFS EBJMZ WPMBUJMJUZ BU UIJT MFWFM XIFSF UIF EFDSFBTF JT WPMBUJMJUZ JT BOE UIF UPUBM PVUTUBOEJOH EFCU JT HS SJTLZ CPSSPXJOH QPTJUJPOT XJMM OPU CF GVMMZ MJRVJEBU XJUI UIF TBNF EBJMZ WPMBUJMJUZ BTTVNQUJPO OPOF PG BOE UIF QSPUPDPM DBO TDBMF UP BU MFBTU Y PG UIF DVSSF

'JHVSF IJHIMJHIUT UIF TBGF PQFSBUJOH TQBDF PG UIF QSPU UIF WPMBUJMJUZ JT CFMPX BOE UIF MJRVJEJUZ QPPM T UP EBJMZ USBEJOH WPMVNF "T UIF WPMBUJMJUZ SFBDIFT TP UIFJS TVQQMJFE BTTFUT

#JU.&9

5IF EBJMZ WPMBUJMJUZ JT DPOWFSUFE GSPN SFBMJ[FE XFFLMZ WPMBU

B 1FSDFOUBHFT PG TJNV>M1BWUJPO MSJVUJEUBBUBFMOE XJIOU

C 1FSDFOUBHFT PG TJNV>M1BWUJPOVOEFS UIPMMFDUFXSJBMJ[FE EFCU

'JHVSF   5IF $PNQPVOE DPOUSBDUT BSF EFQMPZFE XJUI UIF EFGB
XFFLMZ IJTUPSJDBM &5) WPMBUJMJUZ JOEFY   UIF DVSSFOU EBJMZ
DBM EBJMZ WPMBUJMJUZ JT BSPVOE   5IF TJNVMBUJPO BTTVNFT
$PNQPVOE T DVSSFOU UPUBM PVUTUBOEJOH TUBCMFDPJO EFCU W
UBM PVUTUBOEJOH EFCU JT BSPVOE   XIJDI JT UIF UPUBM PVUTU
USBEJOH WPMVNF PG UIF DPMMBUFSBM BTTFU

5PUBM 0VUTUBOEJOH %FCU WT -JRVJEBUJPO *

*OUIJT TFDUJPO XF FYBNJOF IPX DIBOHFT JO MJRVJEBUJPO J
&5) TEBJMZ WPMBUJMJUZ JO SBOHFT CFUXFFO BOE *G
TZOUIFUJD QSJDF USBKFDUPSJFT BOE BEKVTU MJRVJEBUJPO
PG UIF TJNVMBUJPO SVOT FOET XJUI B MJRVJEBUBCMF PS VOE

'JHVSF 1FSDFOUBHFT PG TJN 1MPU GJUUFE SVOT UIBU FOEFE XJUI 5IF DV
GBVMU MJRVJEBUJPO JODFOUJWF JT 5IF TJNVMBUJPO BTTVNF
PG &5) EBJMZ USBEJOH WPMVNF $PNQPVOE T DVSSFOU UPUBM PV
..64% 5IF DVSSFOU UPUBM PVUTUBOEJOH EFCU JT BSPVOE
WBMVF OPSNBMJ[FE CZ UIF EBJMZ USBEJOH WPMVNF PG UIF DPMMB

5P CFUUFS BTTFTT UIF TZTUFN T TBGFUZ XJUI WBSJPVT MJ
WPMBUJMJUZ BT BO BTTVNQUJPO 8JUI UIF EFGBVMU MJRV
HFU MJRVJEBUFE XIFO UIF UPUBM PVUTUBOEJOH EFCU WBMV
BT DBO CF TFFO %JTDVTTJPO HJT XIFO XF FWBMVBUF UIF VOEFS
UIF TBNF SBOHF PG QBSBNFUFST BMM UIF TJNVMBUJPO SVO
(JWFO UIBU B MPX MJRVJEBUJPO JODFOUJWF JT JOFrFDUJWF
NBZ XPOEFS XIZ UIFSF T OP VOEFS DPMMBUFSBMJ[BUJPO SJT
GPS UIJT JT UIBU UIF EFGBVMU DPMMBUFSBM GBDUPS TFUT
QSJDF DIBOHFT PG WPMBUJMJUZ JT JOTVrJDJFOU UP NPWF
EFCU WBMVF

5PUBM 0VUTUBOEJOH %FCU WT $PMMBUFSBM '

'JHVSF DPNQBSFT UIF TZTUFN T TBGFUZ VOEFS EJrFSFOU DPMM
JOH EFCU "T NFOUJPOFE CFGPSF UIF DPMMBUFSBM GBDUPS
SFRVJSFNFOU 5IF DMPTFS UIF DPMMBUFSBM GBDUPS JT UP
UP NBJOUBJO B CPSSPXJOH QPTJUJPO DPOTFRVFOUMZ CPS
USBUFE JO UIF TZTUFN JT SJTLZ JO UIF IJHI DPMMBUFSBM GBD
SFHJPO *EFBMMZ DPMMBUFSBM GBDUPS TFUUJOH TIPVME C
"TTVNJOH B MJRVJEBUPS EPFTO U FYJTU BOE UIF DPMMBUFS
EFGBVMU JG UIF DPMMBUFSBM BTTFU QSJDF FYQFSJFODFE H
TVHHFTUT UIBU UIF QSPUPDPM JT TBGF XIFO UIF DPMMBUFSB
JU T SBSF UP QSJDF EFDMJOF XJUI UIF HJWFO WPMBUJMJUZ BT

$PODMVTJPOT

*O UIJT SFQPSU XF DPOEVDUFE B NBSLFU SJTL BTTFTTNFOU
CBTFE TJNVMBUJPOT SVO BHBJOTU UIF $PNQPVOE DPOUSBDU
BOJTN VOEFS B XJEF SBOHF PG NBSLFU WPMBUJMJUZ BOE TJ
DPM DBO QSFWFOU CPSSPXFST GSPN CFDPNJOH VOEFS DPM
BMTP VTFE IJTUPSJDBM NBSLFU EBUB GSPN DFOUSBMJ[FE D
BTTVNQUJPOT BCPVU WPMBUJMJUZ BOE TMJQQBHF BSF SFQS
8F GPVOE UIBU UIF QSPUPDPM BT DVSSFOUMZ QBSBNFUFS
UP BU MFBTU Y UIF DVSSFOU CPSSPX TJ[F BT MPOH BT &5) QSJ
IJHIT 8F BMTP BOBMZ[FE UIF FrFDUJWFOFTT PG UIF MJRVJEB
UXP QSJNBSZ SJTL MFWFST UIF $PNQPVOE QSPUPDPM FNQM
TBGFUZ BOE DBQJUBM FrJDJFODZ
0VS NFUIPEPMPHZ DBO BMTP CF BQQMJFE UP PUIFS DPMMB
DBOUMZ EJrFSFOU MJRVJEJUZ QSPGJMFT TVDI BT 3&1 5IJT X
IPX UP DIPPTF DPMMBUFSBM GBDUPST BOE MJRVJEBUJPO JOD
UIF QSPUPDPM

B 1FSDFOUBHFT PG TJNV>M1BWUJPBOMSJVOJEUBHBJCFMOE KEJOU

C 1FSDFOUBHFT PG TJNV>M1BWUJPBVOSEFT UDIBMUFBUEFXSJBUM J[FE EFCU

'JHVSF 5IF DVSSFOU EFGBVMU DPMMBUFSBM GBDUPS JT 5IF T
JUZ BOE ..64% PG &5) EBJMZ USBEJOH WPMVNF $PNQPVOE T DV
EFCU WBMVF JT BSPVOE ..64% 5IF DVSSFOU UPUBM PVUTUBOEJO
PVUTUBOEJOH EFCU WBMVF OPSNBMJ[FE CZ UIF EBJMZ USBEJOH W

#JCMJPHSBQIZ

"MBNFEB 3FTFBSDI h*OWFTUJHBUJPO JOUP UIF -FHJUJN
DIBOHF 7PMVNF v +VM 6 3 - ?iiTb,ff7i6X+QKfpQHmK2@`2TQ`i@T T2`XT/7

"MNHSFO 3PCFSU $IFF 5IVN &NNBOVFM)BVQUNBOO BOE)F
PG FRVJUZ NBSLFU JMMJRVJEJUZ 7PM v QQ n

"OHFSJT (VJMMFSNP)TJFO 5BOH ,BP 3FJ $IJBOH $IBSMJF
BOBMZTJT PG 6OJTXBQ NBSLFUT v BS9JW

"SOPUU 3PCFSU % +BTPO)TV BOE 1IJMJQ .PSSJT h"VUPNBUFE
BMZTUT +PVSOBM 7PM QQ n

"ULJOTPO ,FOEBMM BOE 5IPNBT 1PMMPDL 4QFDJBM 'PSN 0OMZ

#JU.&9 h&70- % 8FFLMZ)JTUPSJDBM &UIFS 7PMBUJMJ ?iiTb,ffrrrX
#BiK2tX+QKf TTfBM/2tfX1oPGd.

#JUXJTF "TTFU .BOBHFNFOU h#JUXJTF $SZQUP *OEFY .F ?iiTb,ff
rrrX#BirBb2BMp2biK2MibX+QKfBM/2t2bfK2i?Q/QHQ;v

#PPLTUBCFS 3JDIBSE .BSL 1BEESJL BOE #SJBO 5JWOBO
OBODJBM WVMOFSBCJMJUZ v 0GGJDF PG 'JOBODJBM 3FTFBSDI 8PSLJOH 1BQFS
n

$IJUSB 5 . 2VBJOUBODF 4)BCFS BOE 8 .BSUJOP
#MPDLDIBJO QSPUPDPMT JMMVTUSBUFE *&&& 4ZNQPTJVN PO
TJVN PO 4FDVSJUZ BOE 1SJWBDZ QQ 8PSLTIPQ PO &V4 18
SP418

$IJUSB 5BSVO h$PNQFUJUJWF FRVJMJCSJB CFUXFFO T
QSFQSJOU BS9JW

$VSDVSV 4UFQIBOJF & $IBSMFT 1 5IPNBT 'SBODJT & 8BSOPD
DPWFSFE FRVJUZ QBSJUZ BOE SFCBMBODJOH v *OUFSOBUJPOBM
.POFZ BOE 'JOBODF QQ n

%BJBO 1IJMJQ 4UFWFO (PMEGFEFS 5ZMFS ,FMM :VORJ -J 9
EFOCBDI BOE "SJ +VFMT h'MBTI #PZT 'SPOUSVOOJ
.JOFS &YUSBDUBCMF 7BMVF BOE *OTUBCJMJUZ JO %FDFOUSBMJ[FE
BOE 1SJWBDZ 41 n

&JTMFS ;PMUBO +FBO 1IJMJQQF #PVDIBVE BOE +VMJFO ,P
PSEFS CPPL FWFOUT NBSLFU PSEFS 2VBOUJUBUJWF 'JOBODF
/P QQ n

'BHJPMP (JPSHJP BOE "OESFB 3PWFOUJOJ h.BDSPFDPO
NPEFMT SFEVY /FX EFWFMPQNFOUT +PVSOBM PG "QQMJFE /HFT BIFB

(BUIFSBM +JN h/P EZOBNJD BSCJUSBHF JO UIF JNQMJFE
/P QQ n

(BUIFSBM +JN BOE "MFYBOEFS 4DIJFE h0QUJNBM USBEF
JBO NPUJPO JO UIF "MNHSFO BOE $ISJTT GSBNFXPSL PG 5
"QQMJFE 'JOBODF 7PM /P QQ n

(FBOBLPQMPT +PIO 3PCFSU "YUFMM + %PZOF 'BSNFS 1FUF
(PMETUFJO .BUUIFX)FOESFZ /BUIBO .1BMNFS BOE $IVO
UFNJD SJTL WJB BO BHFOU CBTFE "NFSJDBO &DPOPNJD 3FW
7PM /P QQ n

)BMBK (S[FHPS[h"HFOU CBTFE NPEFM PG TZTUFN XJEF

)BOTPO 3PCJO h$PNCJOBUPSJBM PSEFS BMMPDBUJPO
UJF7SFM /P QQ n

)BXLFT "MBO (BOE %BWJE 0BLFT h" DMVTUFS QSPDFTT
QSPDFTT +PVSOBM PG "QQMJFE 1SPCBCJMJUZ n

)PVHBO .BUUIFX)POH ,JN .JDBI -FSOFS BOE #JUXJTF "TTFU
BOE /PO &DPOPNJD 5SBEJOH JO #JUDPJO &YQMPSJOH UIF 3
%JHJUBM $PNNPEJUJFT "TTFU .BOBHFNFOU

)TV +BTPO $ h$BQ XFJHIUFE QPSUGPMJPT BSF GSBHJMF
NFOU .BOBHFNFOU /P

)VMM +PIO *OUSPEVDUJPO UP GVUVSFT BOE PQUJPOT
/+

,MBHFT .VOEU "SJBI BOE "OESFB .JODB h *O 4UBCJMJU
4QJSBMT BOE 4UBCMF 'JOBODF 9JW

-BXMFS (SFHPSZ ' h4UPDIBTUJD DBMDVMVT JO "DBJO USPE
.BUIFNBUJDBM 4PDJFUZ

-FTIOFS 3PCFSU BOE (FPrSFZ)BZFT h$PNQPVOE 5IF .PO
DBM SFQPSU

-JV "ORJ .BSL 1BEESJL 4UFWF : :BOH BOE 9JOHKJB ;IBOH
BHFOU CBTFE NPEFM BQQSPBDI UP FOEPHFOPVT NPOFZ
OBODF

.BLBSPW *HPS BOE "OUPJOFUUF 4DIPBS h5SBEJOH BOE B
+PVSOBM PG 'JOBODJBM &DPOPNJDT

/ZTUSVQ 1FUFS 4UFQIFO #PZE &SJL -JOETUSÕN BOE)FOSJ
GPMJP TFMFDUJPO XJUI QBSBNFUFS VODFSUBJOUZ 3FWJFX PG
n

0CBEJB "MFYBOEFS h&YQMPSJOH %F'J USBEJOH
%F'J v /PW 63- ?iiTb,ffrrrXi?2#HQ+F+`vTiQX+QKfTQbif98
2tTHQ`BM;@/27B@i` /BM;@bi` i2;B2b@ `#Bi` ;2@BM@/27B

0UINBO "CSBIBN %BWJE.1FOOPDL %BOJFM.3FFWFT BOE 5V
DBM MJRVJEJUZ TFOTJUJWF BSCJUSBHF BOE NBSLFU &DPOPNJDT
QVUBUJPO /P Q

1BT[LF "EBN 4BN (SPTT 'SBODJTDP .BTTB "EBN -FSFS +BN
5SFWPS ,JMMFFO ;FNJOH -JO /BUBMJB (JNFMTIFJO -VDB "
QFSBUJWF TUZMF IJHI QFSGPSNBODF EFFQ MFBSOJOH MJCSBSZ
1SPDFTTJOH 4ZTUFNT

3PVWJFSF 4JNPO EF MB h5PLFOT $VSWFE
$VSBUJPO .BSLFUT v /PW 63- ?iiTb,ffK2/BmKX+QKf!bBKQM/H`
iQF2Mb@k@y@+m`p2/@iQF2M@#QM/BM;@BM@+m` iBQM@K

4DINJEU 5PN h-JRVJEBUPST 5IF 4FDSFU 8IB
'VODUJPO v +BO 63- ?iiTb,ffK2/BmKX+QKf/` ;QM7Hv@`2b2
HB[mB/ iQ`b@i?2@b2+`2i@r? H2b@?2HTBM;@/27B@7mM+iB

4FDVSJUJFT BOE &YDIBOHF $PNNJTTJPO h4FMG 3FHVME
0SEFS %JTBQQSPWJOH B 1SPQPTFE 3VMF $IBOHF BT .PEJGJ
UIF -JTUJOH BOE 5SBEJOH PG 4IBSFT PG UIF #JUXJTF #JUDP
& v 0DU 63- ?iiTb,ffrrrXb2+X;Qpf`mH2bfb`QfMvb2 `+ fkyRNfj9
T/7

5ÓUI #FODF :WFT -FNQFSJFSF $ZSJM %FSFNCMF +PBDIJN %
+ 1 #PVDIBVE h"OPNBMPVT QSJDF JNQBDU BOE UIF DSJU
NBSLFUT 3FWJFX /P Q

8FJ 8BOH $IVO h-JRVJEJUZ BOE NBSLFU FGGJDJFODZ JO D
UFST 7PM QQ n

:BOH 4UFWF .BSL 1BEESJL 3PZ)BZFT "OESFX 5PEE "OESFJ
4DIFSFS h#FIBWJPS CBTFE MFBSOJOH JO JEFOUJGZJOH
*&&& $POGFSFODF PO $PNQVUBUJPOBM *OUFMMJHFODF
$*'&S QQ n *&&&

;IBOH 5BJZBOH h*OUSPEVDJOH ,FFQFS%"0 BO
JUZ VOEFSXSJUFS v +BO ?iiTb,ffK2/BmKX+QKfF22T2`/ Qf
BMi`Q/m+BM;@F22T2`/ Q@ M@QM@+? BM@HB[mB/Biv@mM

1BSU *7

"QQFOEJY

(MPTTBSZ

- s%FCU "NPVOU PG BTTFU CPSSPXFE GSPN BO BTTFU QPPM
- s6OEFS DPMMBUFSBMJ[FE "O BDDPVOU JT VOEFS DPMMBU
  FYDFFET UIF WBMVF PG UIF DPMMBUFSBM
- s$PMMBUFSBM GBDUPS .BYJNVN EFCU UP DPMMBUFSBM S
  UIF EFCU UP DPMMBUFSBM SBUJP FYDFFET UIF DPMMBUF
  MJRVJEBUFE
- s$PMMBUFSBMJ[BUJPO SBUJP 5IF SBUJP PG DPMMBUFSBM
  QPJOUT 'PS JOTUBODF B DPMMBUFSBMJ[BUJPO SBUJP P
  BT NVDI DPMMBUFSBM EFQPTJUFE JOUP UIF DPOUSBDU BT
  DSFUFMZ UIJT XPVME NFBO UIBU POF NVTU EFQPTJU X
  PG B TUBCMFDPJO
- s#PSSPXJOH DBQBDJUZ $VSSFOU WBMVF PG DPMMBUFSB
  CZ UIF DPMMBUFSBM GBDUPS

s $PMMBUFSBM SFRVJSFNFOU 7BMVF PG EFCU EJWJEFE CZ

s -JRVJEBUBCMF "O BDDPVOU JT MJRVJEBUBCMF JG UIF BD
SPXJOH DBQBDJUZ *O PUIFS XPSET BO BDDPVOU JT MJR
WBMVF GBMMT CFMPX UIF DPMMBUFSBM SFRVJSFNFOU

s 4MJQQBHF 5IF BNPVOU PG QSJDF JNQBDU UIBU B MJRVJEB
MBUFSBM 4MJQQBHF JT EFGJOFE BT UIF EJrFSFODF CFUX
NJEQPJOU QSJDF BOE UIF FYFDVUJPO QSJDF 4MJQQBHF JT
q BU UJNF *O UIF DPOUFYU PG UIJT QBQFS p(q; t) = p_NJE(t) − p_FYFD(q; t) 5IJT RVBOUJUZ JT VTVBMMZ B GVO
WBSJBCMFT TVDI BT JNQMJFE BOE SFBMJ[FE WPMBUJMJ
JNQBDU XJUI BDBEFNJD MJUFSBUVSF

*OUFSFTU 3BUF $VSWFT

8JUIJO UIF DSZQUPDVSSFODZ TQBDF NJOJTUJD GVODUJPOT F
TUBUF UIBU EFUFSNJOF CJE BOE BTL TQSFBET XJUIJO MFOEJOH DVS
UIF BMHPSJUINJD HBNF UIFPSZ MJUFSBUVSF BOE XF SFGFS UP UI
BVUPNBUFE NBSLFU NBLFST GJSTU JOUSPEVDFE UP &UIFSFVN
MB 3PVWJFSF UP DSFBUF UPLFOJ[FE NBSLFUT XIPTF CVZ BO
BMHPSJUINJDBMMZ *OTUFBE PG VTJOH B CPOEJOH DVSWF U
QSPUPDPM VUJMJ[FT B CPOEJOH DVSWF UP DPNQVUF UIF TQ
JOUFSFTU SBUFT 0OF UIBU UIJOL PG UIJT BT BO BOBMPHVF PG
BMCFJU DPNQVUFE JO B EJrFSFOU NBOOFS

5IF DPOUSBDU BMTP VTFT UIF CPOEJOH DVSWF UP FOGPSD
TVQQMZ JOUFSFTU SBUF NVTU CF TUSJDUMZ MPXFS UIBO UIF
USVF UIFO BO BSCJUSBHFVS DPVME CSFBL UIF TZTUFN CZ CP
BEEJOH MJFOEFE UPLFOT UP UIF MJRVJEJUZ TVQQMZ MFBEJO
DPOUSBDU BMTP FOGPSDFT TPGUFS DPOTUSBJOUT UIBU DPO
CPSSPXJOH JOUFSFTU SBUFT 5IF NBJO JEFB CFIJOE UIF DVS
NPSF MJRVJEJUZ TVQQMZ UIBO CPSSPXJOH EFNBOE UIFO UIF
CF TJHOJGJDBOUMZ MPXFS UIBO UIF JOUFSFTU SBUF UP CPSS

'PSNBMMZ UIF $PNQPVOE DPOUSBDU DPOUSPMT UIF CPOEJOH DVSWF
PG VUJMJ[BUJPO SBUF [0, 1] *G XF EFOPUF UIF CPSSPXJOH E

)BOTPO
0UINBO FU BM
3PVWJFSF
-FTIOFS BOE)BZFT

IFJHIU JO UPLFOT BT B_t BOE BT UIF MJRVJEJUZ BT L_t TVQQMZ UIF MFWFSBHF [BUJPO S
EFGJOFE BT

$$U_t = \frac{B_t}{L_t + B_t}$$

8F DPNQVUF UIF CPSSPX$_t$ XBJOH JOUFSFTU SBUF VTJOH B QJFDFXJTF MJOFBS
GPSNVMB $_0$, $_1$ XIFSF BSF JOUFSFTU SBUF QBSBNFUFST XIJDI EFUFSNJOF UIF SBUF BOE TQSFBE PG
UIF TQSFBE CFUXFFO TVQQMZ BOE CPSSPX JT EFTDSJCFE $_0$ CZ TQSFBE

$$_t = U_t(_0 + _1U_t)$$
$$`_t = (1 \quad _0) \quad _t$$

'PS SFGFSFODF UIF $PNQPVOE 7$_0$ DPOUSBDU VTFT $_1$ 45 WT 5 UIF WBMV
DIPJDF PG RVBESBUJD CPOEJOH DVSWF IBT B WBSJFUZ PG CFO
PG BSUJDMFT BOE QBQFST

4JNVMBUJPO %FUBJMT

&OWJSPONFOU BOE 4BNQMJOH

5IF TJNVMBUJPO FOWJSPONFOU BMMPXT GPS DPOGJHVSBUJP
JOH BHFOU EJTUSJCVUJPO BHFOU CFIBWJPS BOE TNBSU DPO
NFOU EJSFDUMZ JOUFSBDUT XJUI UIF $PNQPVOE TNBSU DPOU
&UIFSFVN WJSUVBM NBDIJOF "U FBDI UJNF TUFQ BHFOUT PC
BCMFT BOE PO DIBJO DPOUSBDUT 5IF QPMJDZ EFGJOFT UIF B
8IFO FBDI BHFOU QFSGPSNT BO BDUJPO UIF TJNVMBUJPO FO
CMPDLDIBJO BOE VQEBUFT UIF TUBUF PG UIF TNBSU DPOUSBD
'PS FBDI TFU PG QBSBNFUFST XF SVO TJNVMBUJPO UJNF
JT MBSHF FOPVHI UP DPWFS B XJEF SBOHF PG CPSSPXJOH EJT

4JNVMBUJPO %VSBUJPO

*G UIF TJNVMBUJPO UJNF EVSBUJPO JT UPP TIPSU UIF QSJDF
PWFSBMM BHFOU CFIBWJPS 0O UIF PUIFS IBOE FWFO JG XF B
QBSUJDJQBOUT BOE XPO U PGUFO DIBOHF UIFJS EFCU QPTJU
XIFO UIF UJNF EVSBUJPO JT MPOH 5P CBMBODF CPUI PG UIFT
EVSBUJPO UP SVO TJNVMBUJPOT

$IJUSB 0CBEJB

"HFOUT

5IF SF BSF GPVS UZQFT PG BHFOUT JO UIF TJNVMBUJPO TFUVQ
O UIF JOJUJBMJ[BUJPO QIBTF B TVQQMJFS TVQQMJFT %" UP
BOE SFUVSOT D%"* #PSSPXFST TVQQMZ &5) BT DPMMBUFSBM
FBDI UJNF TUFQ MJRVJEBUPST MPPLT GPS VOEFS DPMMBUFSB
UIF SFTVMUJOH &5) DPMMBUFSBM QPTJUJPO 5IF QSJDF PSBD
PO UIF (#. NPEFM PS UIF QSJDF USBKFDUPSJFT GSPN IJTUPSJ

1SJDF 5SBKFDUPSZ

8F VTF B TUBOEBSE (FPNFUSJD #SPXOJBO NPUJPO (#. UP T
TUPDIBTUJD QSPDFTT PCFZT UIF ²UdPX$_t$=UFSdIBTU$_t$dM$_t$ EJrFSFOUJ
XIFdSF$_t$ JT UIF TUBOEBSE 8RFO(#.JT NBMTP FRVOWBMFOU UP UIF FY
SBOEPNMZ WBSZJOH RVBOUJUZ X$_t$ = GPX$_0$ 2tP XT B$\frac{2}{2}$# S+P XQ$_t$ JBO NPUJP
5IF HSBQI CFMPX TIPXT UIF &5) QSJDF USBKFDUPSJFT XJUI B
WPMBUJMJUJFT 'PS FBDI WPMBUJMJUZ XF HFOFSBUF EJrF
WBSJBUJPO
  8F BMTP TJNVMBUFE PUIFS -ÊWZ QSPDFTTFT JODMVEJOH Q
DSBTI)FPTXFWFS JG UIF KVNQ QBSBNFUFST F H UIF NFBO PG B
UP UIF BWFSBHF KVNQ CFJOH MBSHFS UIBO UIF DPMMBUFSBM
KVNQ QBSBNFUFST JT TNBMMFS UIBO UIF DPMMBUFSBM GBDUP
TJNVMBUJPO SFTVMUT XJUI BOE XJUIPVU KVNQT 'VUVSF XPS
NFNPSZ TVDI BT)BXLFT QSPDFTTFT IBWF RVBMJUBUJWFMZ EJ

4MJQQBHF

5IF TRVBSF SPPU NPEFM JT UIF NPTU QPQVMBS NBSLFU JNQB
WBMJEBUF XIFUIFS UIF TBNF BTTVNQUJPO IPMET JO DSZQUP
FYDIBOHF PSEFS CPPL TOBQTIPU CFUXFFO UP GSPN $P
XJUI FYUSFNFMZ MPX QSJDFT XJMM CJBT UIF TMJQQBHF FTUJ
UIF NBSLFU NJE QSJDF XFSF GJMUFSFE PVU QSJPS UP GJUUJ
TMJQQBHF JT FTUJNBUFE CZ UIF NBSLFU NJE QSJDF NJOVT U

/PUF UIBU GPS TFNJ NBSUJOHBMFT BOE OPO QSFEJDUBCMF QSPDF
QBSBNFUSJ[BCMF GBNJMZ PG QSPDFTTFT UIBU DBO CF XSJUUFO BT UIF TV
KVNQ QSPDFTT 4FF UIF -ÊWZ ²UP -BXMFS GPS BEEJUJPOBM EFUBJMFE EJTDVTTJPO
)BXLFT BOE 0BLFT
"MNHSFO FU(BM UIFSBM BOE 4DIJFE

'JHVSF 4JNVMBUFE QSJDF USBKFDUPSJFT VOEFS EJrFS

EFGJOF UIF TMJQQBHF SBUJP UP CF UIF TMJQQBHF OPSNBMJ[
TMJQQBHF FTUJNBUF IFSF JT NPEFMJOH UIF XPSTU DBTF TDF
BSF HSFFEZ *O QSBDUJDF B USBEFS XJUI B MPOHFS UJNF QS
05$ NBSLFU PS TQMJU POF PSEFS JOUP NVMUJQMF TVCPSEFST

8F VTF B OPO MJOFBS MFBTU TRVBSFT NFUIPE UP DIPPTF UIF G
GVODUJPOT UP SFQSFTFOU PVS TMJQQBHF NPEFM

s 4RVBSF SPPU QSJDF TMJQQBHF JT QSPQPSUJPOBM UP UI

p(q) = I p q

s -JOFBS QSJDF TMJQQBHF JT QSPQPSUJPOBM UP UIF RVBO p(q) = Iq

"ULJOTPO BOE)BO

s2VBESBUJD QSJDF TMJQQBHF JT QSPQPSUJPOBM UP UIF p(q) = OBM UP UIF
I q^2

XIFSF UIF JOUFOTJUZ UBOU EFUFSNJOFE CZ UIF NPEFM BOE
CZ UIF BWFSBHF EBJMZ USBEJOH WPMVNF 5IFPSFUJDBMMZ
DIPTFO UP SFEVDF UIF NPEFM TFSSPS 'PS UIF TBLF PG JOUFS
XF DIPTF TJNQMF GVODUJPOBM GPSNT GPS UIJT BOBMZTJT

'JHVSF)JTUPSJDBM PSEFS TJ[F WFSTVT FTUJNBUFE QSJDF TM

0VS SFTVMUT GSPN GJUUJOH R^2 = 0:76 UIF EBUB BSF QSFTFOUFE
UIF TRVBSF SPPU NPEFM R^2 = 0:56 XF QJDLFE UIF MJOFBS NBSLFU JNQ
TJNVMBUJPO .PSFPWFS UIF MJOFBS $5M,000 QBSU NPEFM VTFE
JO 'JHVSF 0OF SFBTPO GPS XIZ UIF TRVBSF SPPU NPEFM EPFT O
JT B EFBSUI PG MJRVJEJUZ $SZQUPDVSSFODZ FYDIBOHFT

UIF MJRVJEJUZ UIBU FYJTUT PO FYDIBOHFT JO USBEJUJPOBM
PG DIBOHF JO TMJQQBHF XJUI SFTQFDU UP DIBOHF JO PSEFST
JODSFBTFT (E) JT DPODBWF)PXFWFS XIFO UIFSF T OPU FOPV
XJMM CF NBHOJGJFE XIFO UIF PSEFS TJ[F JT MBSHF

#PSSPXFS

5IF CPSSPXFS T DPMMBUFSBMJ[BUJPO SBUJP WBMVF PG DPM
UPUBM DPMMBUFSBM WBMVF BSF UXP JOGMVFOUJBM QBSBNF
XPSME $PNQPVOE CPSSPXFS EBUB UP CFUUFS VOEFSTUBOE I
JO 'JHVSF XT UIF KPJOU EJTUSJCVUJPO PG UIF MPHBSJUIN PG
DPMMBUFSBMJ[BUJPO SBUJP "HBJO UIF DPMMBUFSBM GBDU
WBMVF UIBU DBO CF CPSSPXFE TP UIBU UIF JOWFSTF PG DPM
J[BUJPO SBUJP UIBU B CPSSPXFS XJMM OPU HFU MJRVJEBUFE

'JHVSF $PNQPVOE CPSSPXFS EBUB GSPN DPOUSBDU JODFQUJPO
MJOF SFQSFTFOUT UIF MJRVJEBUJPO UISFTIPME F H UIF NJOJNVN

5P DMPTF UIF HBQ CFUXFFO TJNVMBUJPO BOE SFBMJUZ XF
FSBM WBMVF BOE DPMMBUFSBM J[BUJPO SBUJP GSPN UIF SFB
HFOFSBUJOH B SFQSFTFOUBUJWF TBNQMJOH PG CPSSPXFST

s -PBE $PNQPVOE T CPSSPXFS EBUB

- Use multivariate kernel density estimation to fit the borrower data. The probability density contour plot is on the above right-hand side.
- Sample the collateral value and collateralization ratio from the probability density contour plot.
- During sampling, it's possible that some sampled data points are under-collateralized. For those data points, increase the collateralization ratio to the minimum value satisfying collateral factor constraint to avoid smart contract initialization failure.
- Re-scale the samples such that the sum of the total outstanding debt value equals the simulation input and uses the processed sample data for simulation.

11.5 Liquidity Supply

In the simulation, a supplier supplies DAI to the protocol, which begins accumulating interest rate. The supplier receives a quantity of cDAI equal to the underlying DAI supplied, divided by the exchange rate. The purpose of having a supplier is to provide liquidity to the system and enabling borrowers to withdraw DAI from the liquidity pool. The DAI supplied amount in the simulation equals the sum of all the borrowers' total outstanding debt + 1.

11.6 Liquidator

A rational liquidator's main goal is to maximize profits, by ensuring that the revenue received from the liquidation incentive outweighs the costs. The slippage model assumes that a liquidator will submit a market order on a single exchange, so the cost is the worst-case estimate. Let the liquidation incentive be denoted by , the trading fee denoted by , and the transaction fee denoted by . We can then formulate the profit p of each trade as

$$p = h9^2((\qquad p(q) \qquad)\underline{q} \qquad ; 0)$$

For each liquidation opportunity, the liquidator repays the minimum value among

- Maximum repay value: borrower's outstanding debt close factor
- Value of borrower's liquidatable collateral
- Liquidator agent's perceived optimal repay amount

The Compound protocol defines maximum repayment amount for liquidating a borrower. If the collateral price drops too fast during periods of extreme volatility and falls below the maximum repay value, a liquidator can only repay up to the value of the borrower's liquidatable collateral. A liquidator also estimates the perceived slippage and calculates the optimal repay amount to maximize the profit. The optimal repayment amount calculation is discussed in the next section. After the liquidator acquires the collateral from the Compound protocol, she immediately sells all received ETH in exchange for USD on an open exchange to realize profit.

11.6.1 Optimal Liquidation Amount

To derive the optimal repayment amount to maximize liquidator profit, we first plug the slippage model into the profit function. For instance, in the linear slippage case, we have the following:

$$p = (\quad I \; \underline{q} \quad) \underline{q}$$

To maximize profit, we find the derivative of profit with respect to the normalized order size $\underline{q}$ and find a value $\underline{q}^*$ such that $\partial_{\underline{q}} p(\underline{q}^*) = 0$. By construction, the value $\underline{q}^*$ is the optimal order size that maximizes the net profit. Performing this calculation yields that the optimal value of $\underline{q}$ to maximize profit is,

$$\underline{q}^* = \frac{\quad}{I}$$

Figure 10 shows liquidation incentive, profit curve and the optimal repay value under various volatility assumptions. The area enclosed by rectangle with vertices $(0,0)$, $(p(\underline{q}),0)$, $(\underline{q}, p(\underline{q}))$, $(0, p(\underline{q})) \in \mathbb{R}^2$ is the liquidator's net profit. Based on the above derivation, we see that a liquidator has no incentive to liquidate any collateral larger than $\underline{q}^*$. If the value of the borrower's liquidatable collateral is less than $\underline{q}^*$, the rational strategy for a liquidator is to liquidate maximum liquidatable collateral.

11.6.2 Compound liquidator time preferences

To be safe, we always assume that liquidators immediately sell their collateral. This may seem like a presumptuous simplification, but it is the expected outcome of any profit-maximizing strategy, regardless of your valuation of the collateral asset.

Let's say that all participants in Compound have a valuation of the collateral asset v. We'll denote the liquidation incentive as $I = 1.05$. Consider a case where there is a collateral position with $75 of DAI borrowed against $101 of ETH. The collateral factor for ETH is .75, so if the price drops by more than $1, the position can be liquidated. The liquidatable position can be then claimed for $95. There are three cases for the liquidator here:

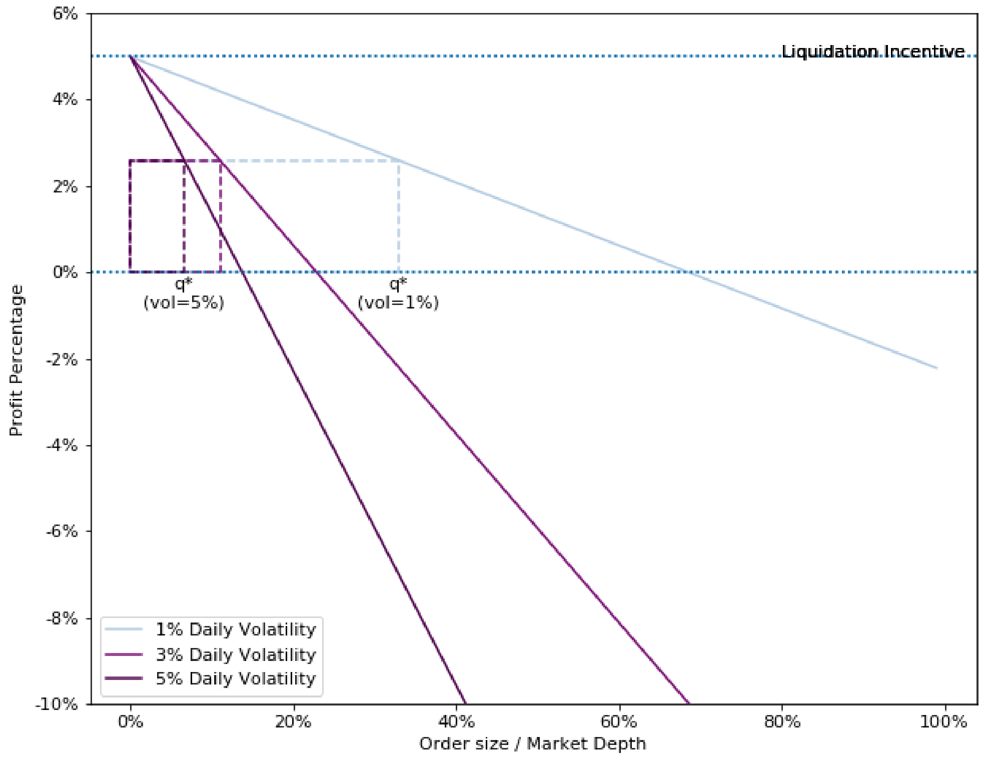

Figure 10: Profit curve and optimal liquidation amount with the assumption of linear slippage model

- **Case 1:** V $100
  In this case, this participant would have bought ETH already, regardless of the liquidation. Assuming there is a large, liquid market for ETH, they will have no more assets to allocate to ETH before the liquidation even occurs.

- **Case 2:** V is between $95 and $100
  In this case, the participant will by the ETH from the liquidation. However, since the market price is greater than V, they will also sell immediately.

- **Case 3:** V < $95
  This participant will still not want to buy the ETH from the liquidation, since the liquidation incentive discount still doesn't give them an opportunity to buy at a price below

their validation.

This simple analysis excludes transaction costs, and the market impact of buying and selling the collateral, but it serves to show that the assumption that liquidators always sell the collateral immediately isn't a glib one. The only case where a rational liquidator would hang on to the collateral is in Case 1, but this implies that they acted irrationally in the past and didn't buy the asset on the open market when they had the opportunity to do so for less than their intrinsic valuation.

11.7 Raw data

	ETH Daily Volatility									
Total Outstanding Debt	0:03	0:1	0:15	0:2	0:25	0:3	0:35	0:4	0:45	0:5
3:0	0:0	0:03	0:03	0:03	0:1	0:17	0:23	0:23	0:27	0:3
2:5	0:0	0:03	0:03	0:03	0:1	0:13	0:23	0:23	0:23	0:27
2:0	0:0	0:0	0:03	0:03	0:1	0:1	0:2	0:23	0:23	0:27
1:5	0:0	0:0	0:03	0:03	0:07	0:13	0:2	0:23	0:23	0:27
1:0	0:0	0:0	0:0	0:03	0:07	0:1	0:13	0:2	0:2	0:27
0:75	0:0	0:0	0:0	0:0	0:07	0:1	0:1	0:17	0:2	0:23
0:5	0:0	0:0	0:0	0:0	0:03	0:07	0:1	0:1	0:2	0:23
0:25	0:0	0:0	0:0	0:0	0:0	0:0	0:0	0:0	0:1	0:13

Table 1: Raw data of figure 4a

	ETH Daily Volatility									
Total Outstanding Debt	0:03	0:1	0:15	0:2	0:25	0:3	0:35	0:4	0:45	0:5
3:0	0:0	0:0	0:0	0:0	0:0	0:0	0:07	0:17	0:2	0:27
2:5	0:0	0:0	0:0	0:0	0:0	0:0	0:03	0:17	0:2	0:23
2:0	0:0	0:0	0:0	0:0	0:0	0:0	0:03	0:13	0:2	0:23
1:5	0:0	0:0	0:0	0:0	0:0	0:0	0:03	0:1	0:13	0:23
1:0	0:0	0:0	0:0	0:0	0:0	0:0	0:0	0:07	0:1	0:2
0:75	0:0	0:0	0:0	0:0	0:0	0:0	0:0	0:03	0:07	0:17
0:5	0:0	0:0	0:0	0:0	0:0	0:0	0:0	0:0	0:1	0:17
0:25	0:0	0:0	0:0	0:0	0:0	0:0	0:0	0:0	0:07	0:1

Table 2: Raw data of figure 4b

Total Outstanding Debt	Liquidation Incentive									
	1:01	1:02	1:03	1:04	1:05	1:06	1:07	1:08	1:09	1:1
3:0	0:27	0:2	0:1	0:07	0:03	0:03	0:03	0:03	0:03	0:03
2:5	0:27	0:13	0:1	0:03	0:03	0:03	0:03	0:03	0:03	0:03
2:0	0:23	0:1	0:03	0:03	0:03	0:03	0:03	0:03	0:03	0:03
1:5	0:23	0:07	0:03	0:03	0:03	0:03	0:03	0:03	0:0	0:0
1:0	0:1	0:03	0:03	0:03	0:03	0:0	0:0	0:0	0:0	0:0
0:75	0:1	0:03	0:03	0:0	0:0	0:0	0:0	0:0	0:0	0:0
0:5	0:03	0:03	0:03	0:0	0:0	0:0	0:0	0:0	0:0	0:0
0:25	0:03	0:0	0:0	0:0	0:0	0:0	0:0	0:0	0:0	0:0

Table 3: Raw data of figure 5

Total Outstanding Debt	Collateral Factor								
	0:5	0:55	0:6	0:65	0:7	0:75	0:8	0:85	0:9
3:0	0:03	0:03	0:03	0:03	0:03	0:03	0:1	0:13	0:2
2:5	0:03	0:03	0:03	0:03	0:03	0:03	0:03	0:1	0:17
2:0	0:03	0:03	0:03	0:03	0:03	0:03	0:03	0:1	0:17
1:5	0:0	0:03	0:03	0:03	0:03	0:03	0:03	0:07	0:17
1:0	0:0	0:0	0:0	0:0	0:0	0:03	0:03	0:07	0:1
0:75	0:0	0:0	0:0	0:0	0:0	0:0	0:0	0:03	0:07
0:5	0:0	0:0	0:0	0:0	0:0	0:0	0:0	0:03	0:03
0:25	0:0	0:0	0:0	0:0	0:0	0:0	0:0	0:03	0:03

Table 4: Raw data of figure 6a

Total Outstanding Debt	Collateral Factor								
	0:5	0:55	0:6	0:65	0:7	0:75	0:8	0:85	0:9
3:0	0:0	0:0	0:0	0:0	0:0	0:0	0:0	0:03	0:1
2:5	0:0	0:0	0:0	0:0	0:0	0:0	0:0	0:03	0:07
2:0	0:0	0:0	0:0	0:0	0:0	0:0	0:0	0:0	0:07
1:5	0:0	0:0	0:0	0:0	0:0	0:0	0:0	0:0	0:1
1:0	0:0	0:0	0:0	0:0	0:0	0:0	0:0	0:0	0:03
0:75	0:0	0:0	0:0	0:0	0:0	0:0	0:0	0:0	0:03
0:5	0:0	0:0	0:0	0:0	0:0	0:0	0:0	0:0	0:0
0:25	0:0	0:0	0:0	0:0	0:0	0:0	0:0	0:0	0:0

Table 5: Raw data of figure 6b

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